The correlations and anticorrelations in QPO data

M. A. Abramowicz[1], D. Barret[2], M. Bursa[3], J. Horák[3], W. Kluźniak[4], P. Rebusco[5], G. Török[6]

[1] Department of Physics, Göteborg University, S 412 96 Göteborg, Sweden
[2] Centre d'Etude Spatiale des Rayonnements, CNRS/UPS, 9 Avenue du Colonel Roche, 31028 Tolouse Cedex 04, France
[3] Astronomical Institute, Academy of Sciences, Boční II 1401, 141 31 Praha 4, Czechia
[4] Max-Planck-Institute für Astrophysik, D-85741 Garching, Germany
[5] Zielona Góra University, Lubuska 2, 65-265 Zielona Góra, Poland
[6] Institute of Physics, Silesian University in Opava, Bezručovo nám. 13, 746 01 Opava, Czechia

Abstract. Double peak kHz QPO frequencies in neutron star sources varies in time by a factor of hundreds Hz while in microquasar sources the frequencies are fixed and located at the line $\nu_2 = 1.5\nu_1$ in the frequency-frequency plot. The crucial question in the theory of twin HFQPOs is whether or not those observed in neutron-star systems are essentially different from those observed in black holes. In black hole systems the twin HFQPOs are known to be in a 3:2 ratio for each source. At first sight, this seems not to be the case for neutron stars. For each individual neutron star, the upper and lower kHz QPO frequencies, ν_2 and ν_1, are linearly correlated, $\nu_2 = A\nu_1 + B$, with the slope $A < 1.5$, i.e., the frequencies definitely are not in a 1.5 ratio. In this contribution we show that when considered jointly on a frequency-frequency plot, the data for the twin kHz QPO frequencies in several (as opposed to one) neutron stars uniquely pick out a certain preferred frequency ratio that is equal to 1.5 for the six sources examined so far.

Key words: QPOs – Neutron stars

1. Introduction

It has been recognized some time ago (Bursa 2002) that the pairs of high-frequency QPOs observed in *all* neutron star sources can be fitted by a single linear relation

$$\nu_2 = 0.89\,\nu_1 + 375\,\text{Hz}\,, \tag{1}$$

where ν_2 and ν_1 refers to the upper and lower observed kHz QPO frequency. The linear correlation between the pair frequencies becomes even more apparent if we restrict ourselves to individual sources, though the coefficients of the relation (1) slightly differ from source to source. A particular example is the case of Sco X-1, where the observed frequencies are well fitted with the linear dependence $\nu_2 = (3/4)\,\nu_1 + 450\,\text{Hz}$ with the accuracy better than one percent.

Abramowicz et al. (2003) pointed out that the distribution of frequency ratios is strongly peaked near the 1.5 value, but made no statement about the actual correlation of frequencies. A concern has been raised, whether their result is a statistical illusion (Belloni et al. 2005).

Correspondence to: horak@astro.cas.cz

In this contribution we examine the linear frequency–frequency correlations for six neutron-star sources: 4U 1820−30, 4U 1728−34, 4U 0614+09, 4U 1608−52, 4U 1735−44 and 4U 1636−53, and show that within errors the correlations are consistent with the following statement: the plots of the six $\nu_2 = A\nu_1 + B$ relations intersect in one point $[N_1, N_2]$, with $N_1 \approx 600\,\text{Hz}$, and the ratio of the frequencies in the intersection point is $N_2/N_1 = 1.5$ to high accuracy. To demonstrate this, we plot the five pairs of coefficients, A and B, and show that they are linearly anticorrelated.

2. Anticorrelation between the slope and shift

We examine the six individual neutron star sources by fitting each of them with a linear formula

$$\nu_2 = A\,\nu_1 + B\,, \tag{2}$$

where the coefficients A and B are referred to as the slope and shift, respectively. The resulting values of the slope and shift and of corresponding errors for each source are summarized

Table 1. Best linear fits and their errors for the frequency–frequency correlation.

Source	A	ΔA	B [Hz]	ΔB [Hz]
4U 1636	0.58	0.03	622	27
4U 1608	0.76	0.03	456	16
4U 1820	1.02	0.09	255	66
4U 1735	0.61	0.05	593	39
4U 0614	0.79	0.02	420	5
4U 1728	0.98	0.02	330	5

in Table 1 and plotted in Figure 1 showing the slope–shift plane. The dependence $A = A(B)$ strongly suggests that the two quantities are anticorrelated. The linear fit for the anti-correlation gives

$$A = (1.50 \pm 0.03) - (0.0016 \pm 0.0001)\,\mathrm{Hz}^{-1}\,B\,. \tag{3}$$

As it was mentioned above, this result is consistent with the statement that the six linear plots of the ν_2–ν_1 relations intersect in one point $[N_2, N_1]$. The frequencies N_2 and N_1 can be determined from the coefficients of the anticorrelation (3), from which we obtain

$$N_2 = (940 \pm 80)\,\mathrm{Hz}\,, \quad N_1 = (625 \pm 40)\,\mathrm{Hz}\,. \tag{4}$$

The ratio of frequencies N_2/N_1 at the intersection point equals to 1.5 with the accuracy of 15%. Hence, it is remarkable that neutron stars QPOs pick up the same 3:2 ratio common for the black-hole sources. We can rephrase the equation (3) to the form

$$A = \frac{3}{2} - \frac{B}{625\,\mathrm{Hz}}\,. \tag{5}$$

The data points of Figure 1 have been obtained through a shift-and-add technique, as described in Barret et al. (2005) applied to the whole archival RXTE data available to date. The technique relies firstly on estimating the QPO frequency down to the shortest timescales permitted by the data statistics, and secondly on identifying the QPOs for which the shift-and-add can be applied (generally the lower QPOs). Having a set of frequencies, the frequencies are then binned (typically with a bin of 10 to 20 Hz), and the individual PDS shifted to the mean frequency within the bin. The averaged PDS so-obtained is then searched for QPOs, and when both QPOs are present, their frequencies are obtained with a Lorentzian fit. A linear fit is then applied to the lower and upper QPO frequencies so obtained. This method has been applied to 4 sources shown in Figure 1 (4U1820; 4U1608, 4U1735, 4U1636), whereas for 0614 and 1728, the linear fit has been applied to the frequencies measured with a shift and add technique applied to a continuous segment of observation (of typical duration close to the orbital period of the RXTE spacecraft). Both methods should yield consistent results: the smaller error bars the points of 0614 and 1728 can be explained by a larger number of frequencies involved in the linear fit. A more careful comparison of the results of the two methods (with estimates of the potential biases, such as only the narrower QPOs are detected, hence with small errors on the frequency determination) is clearly required and will be part of a forthcoming paper (Barret et al. 2005b in preparation).
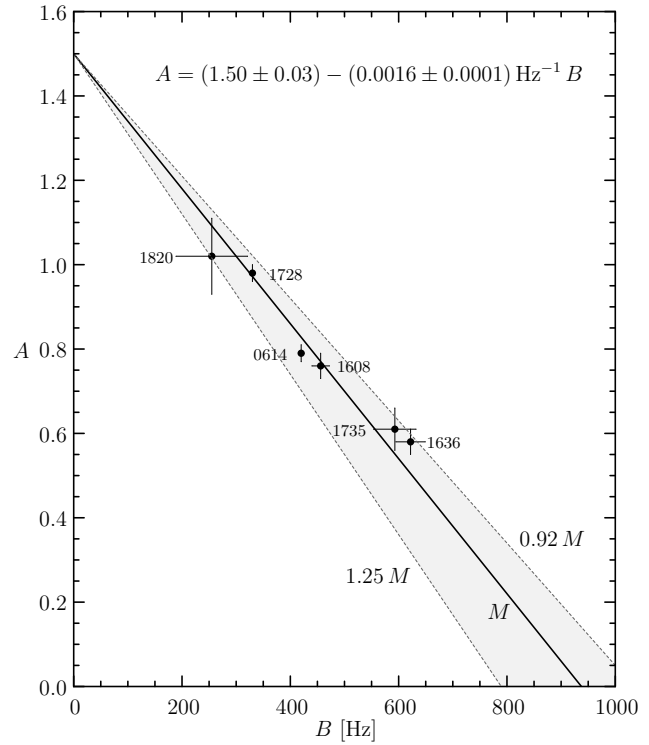

Fig. 1. The anticorrelation between the shift and slope and the effect of $1/M$-scaling. The figure shows the result obtained from the analysis of the data. Each point in the plot corresponds to the particular source. The shift A and the slope B correspond to the best linear fit of the ν_1–ν_2 correlation. Clearly, A and B are anticorrelated among the sources with the best fit (*solid line*) given by equation (3). In the spirit of $1/M$ scaling, it corresponds to a 'typical' neutron-star mass between the examined system. Dashed lines represent the best fit recalled to $0.92M$ and $1.25M$. The value of the fit crosses vertical axis very close 1.5, which is in agreement with the idea that the excitation mechanism of QPOs is the 3:2 resonance.

3. A possible explanation

It has been suggested that QPO arises from a resonant interaction between the radial and vertical oscillation modes in relativistic accretion flows. The strongest possible resonance occurs at the radius, where the radial and vertical epicyclic frequencies are in the 3:2 ratio.

In general, the frequency and the amplitude of non-linear oscillations are not independent. In the lowest order of approximation the observed frequencies ν differ from the eigenfrequencies ν_0 of oscillators by corrections proportional to the squared amplitudes. We consider system having two oscillation modes, whose eigenfrequencies are $\nu_{1,0}$ and $\nu_{2,0}$. The frequencies of non-linear oscillations may be written in the form

$$\nu_1 = \nu_{1,0} + \Delta\nu_1\,, \quad \nu_2 = \nu_{2,0} + \Delta\nu_2\,. \tag{6}$$

The frequency corrections $\Delta\nu_1$ and $\Delta\nu_2$ are proportional to the squared amplitudes

$$\Delta\nu_1 = \kappa_1 a_1^2 + \kappa_2 a_2^2\,, \quad \Delta\nu_2 = \lambda_1 a_1^2 + \lambda_2 a_2^2\,, \tag{7}$$

where κ_1, κ_2, λ_1 and λ_2 are constants depending on non-linearities in the system.

Let us suppose that for some reason the two amplitudes a_1 and a_2 are correlated. Thus, one may consider the amplitudes as functions of a single parameter s,

$$a_1 = a_1(s), \quad a_2 = a_2(s). \tag{8}$$

Such relations can be considered as a natural consequence of an interplay between the resonance excitation mechanism and the dissipation of the energy in the system.

It follows that the frequencies of non-linear oscillations ν_1 and ν_2 are correlated as well. Up to the linear order in s, we obtain from equation (6)

$$\nu_1 = \nu_{1,0} + \nu_{1,0} F_1 s + \mathcal{O}(s^2), \tag{9}$$
$$\nu_2 = \nu_{2,0} + \nu_{2,0} F_2 s + \mathcal{O}(s^2), \tag{10}$$

where the coefficients F_1 and F_2 are given in terms of constants κ_1, κ_2, λ_1 and λ_2 of frequency corrections (7) and of the derivatives $\mathrm{d}a_1/\mathrm{d}s$ and $\mathrm{d}a_2/\mathrm{d}s$ at the point $s = 0$.

Isolating the parameter s from equations (9) and (10) we get the linear correlation between the observed frequencies

$$\nu_2 = A\nu_1 + B, \tag{11}$$

where the slope and shift are respectively given as

$$A = \frac{\nu_{2,0}}{\nu_{1,0}} Q, \tag{12}$$
$$B = \nu_{2,0}(1 - Q) \tag{13}$$

and we define Q as $Q \equiv F_2/F_1$.

The observed frequencies ν_1 and ν_2 of systems with different amplitude prescription (8) are correlated in a different way. Any particular value of Q leads to particular values of the slope and the shift. However, if the eigenfrequencies of the systems are similar, the slope and shift are necessarily anticorrelated. Solving the equations (12) and (13) for the parameter Q, we obtain

$$A = \frac{\nu_{2,0}}{\nu_{1,0}} - \frac{1}{\nu_{1,0}} B. \tag{14}$$

From the resonance condition it follows that the eigenfrequency ratio $\nu_{2,0}/\nu_{1,0}$ is approximately 3/2. Therefore we arrive at

$$A = \frac{3}{2} - \frac{1}{\nu_{1,0}} B. \tag{15}$$

4. Discussion and Conclusions

In black-hole sources the observed QPO frequencies are fixed and always have the ratio 3/2. It has been recognized that their actual frequencies scales inversely with mass M assuming a similar value of the spin (McClintock & Remillard 2005, [van der Klis], [Török][1]). In neutron-star sources the frequencies are not fixed, but their distribution seems to cluster around a single line for each individual source. By linear fitting the observed data, we have found out that these lines intersect around a single point $[N_1, N_2]$, which have coordinates given by equation (4). The fact that the frequencies

are close to 3:2 ratio supports the idea that there is a similar mechanism at work in both classes of sources. Moreover, if we extend the black-hole $1/M$ scaling law up to the frequency of the intersection point, we obtain a mass of order of one solar mass (assuming zero angular momentum for neutron stars), which provides an additional hint.

Assuming that the $1/M$-scaling can be adopted also to the neutron-star QPOs, the fact that the individual positions of sources in the A–B plane do not strictly follow the anticorrelation line can be attributed to small differences in neutron-star masses. By scaling the 614 Hz frequency of equation (3), we find that the A–B is steeper or softer for more massive or less massive sources, respectively. This is demonstrated by the shaded region in Figure 1. Under this assumption the deviation in the masses of examined neutron stars should not be greater than $\sim 20\%$.

References

Abramowicz, M.A., Bulik, T., Bursa, M., Kluźniak, W.: 2003, A&A, 404, L21

Barret, D., Olive, J.-F., Miller, M.C.: 2005, MNRAS, 361, 855

Bursa, M.: 2002, unpublished

Belloni, T., Méndez, M., Homan, J.: 2005, A&A, 437, 209

McClintock, J.E., Remillard, A.R.: 2005, in: W.H.G. Lewin, M. van der Klis (eds.), *Compact Stellar X-ray Sources*, Cambridge Univ. Press, Cambridge (astro–ph/0306213)

---

[1] Articles by other authors in this Volume.